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Macroscopic Deviations from Coulomb's Law without a Photon Mass

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Consideration of a general power law, rather than a Yukawa form, for the electrostatic potential suggests a generalization of Maxwell's equations containing no dimensional parameters. There may be deviations from Coulomb's law even though plane waves propagate in vacuum without dispersion. In considering possible macroscopic deviations from Maxwellian electrodynamics, one must bear in mind the fact that there are different and inequivalent ways to parametrize deviations.

Modern discussions of possible macroscopic deviations from Coulomb's law have generally considered as an alternative the possibility that the quantum of the electromagnetic field might have a nonvanishing rest mass (Goldhaber and Nieto, 1971; Murphy and Burman, 1978). Maxwell's equations would then be replaced by those of Proca (1936). For a point charge Q at rest, these yield the Yukawa expression for the electrostatic potential, $V=(Q/r)\exp(-r/d)$, where d is related to the photon rest mass m by $d=\hbar/mc$. Electromagnetic waves in vacuum would then have the dispersion relation

$$\omega = c(k^2+1/d^2)^{1/2} \tag{1}$$

With this assumption, classical electrostatic experiments, observations of magnetostatic phenomena (such as planetary magnetic fields), or data on wave propagation can be used to set limits on the parameter m. If m is sufficiently small, making d large in comparison with terrestrial distances, then laboratory experiments will no longer be helpful. In fact, observations of astrophysical phenomena can be used to give the limit $d \geq 100$ pc, corresponding to $m \leq 10^{-58}$ g (Murphy and Burman, 1978).

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The addition of a nonvanishing photon mass term seems the most natural way to extend Maxwell's theory in the context of our modern understanding of quantum field theory. But this is, of course, not the only way to account for deviations from Coulomb's law should they be found. In particular, it is possible to find a modification of Maxwell's theory in which plane waves propagate without dispersion in vacuum, but in which there is a deviation from the inverse-square law. One such theory can be developed from an elementary model which Maxwell (1891) used to discuss the experimental evidence for Coulomb's law.

Instead of the Yukawa form for the potential, we consider the expression

$$V = Cr^{-(1+\alpha)} \tag{2}$$

For $\alpha = 0$ this will of course become the usual expression for the Coulomb potential.

We can find a partial differential equation satisfied by (2) by calculating ∇V and $\nabla^2 V$ and eliminating C and r among these expressions. The result is a nonlinear generalization of the Laplace equation,

$$V\nabla^2 V = [\alpha/(1+\alpha)](\nabla V)^2$$

which can also be written as

$$V\nabla \cdot \boldsymbol{E} = [-\alpha/(1+\alpha)]E^2 \tag{3}$$

The latter form suggests an extension to an analogue to the full set of Maxwell's equations. If the field $F_{\mu\nu}$ and the potential A_μ are related in the usual way, the simplest such extension would seem to be

$$F^{\mu\nu}_{,\nu} = -\beta A^\mu (F_{\sigma\tau}F^{\sigma\tau})/(A_\rho A^\rho) \tag{4}$$

$A^0 = V$, $F_{\sigma\tau}F^{\sigma\tau} = 2(B^2 - E^2)$, and $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ with the metric $+ - - -$. The α and β are related by $\beta = \alpha/2(1+\alpha)$. There is no claim that this is a unique form reducing to (3) for static fields.

It is instructive to compare (4) with the Proca system, which, with the same relation between A_μ and $F_{\mu\nu}$, can be written

$$F^{\mu\nu}_{,\nu} = A^\mu/d^2 \tag{5}$$

Equation (5) has the immediate consequence that $A^\mu_{,\mu} = 0$, so that we obtain the wave equation $A^{\mu\nu}_{,\nu} + A^\mu/d^2 = 0$. On the other hand, taking the divergence of (4) yields the more complex condition $(KA^\mu)_{,\mu} = 0$, where K is the invariant $F_{\sigma\tau}F^{\sigma\tau}/A_\rho A^\rho$.

With the Proca system there would be deviations from Maxwell's theory for phenomena on scales of length significantly larger than d. The situation is different for the system (4). If we denote by L a characteristic distance within which A_μ varies significantly, then from (4) we can write the order-of-magnitude equation

$$F^{\mu\nu}_{,\nu} = -A^\mu/(L^2/\beta)$$

In this case the scale of length at which non-Maxwellian features would be seen is $L/|\beta|^{1/2}$. This is no longer a constant, but a length dependent upon the variation of A_μ itself. If $|\alpha| \ll 1$, so that Coulomb's law is a good approximation, then also $|\beta| \ll 1$. Then this length $L/|\beta|^{1/2}$ will always be much greater than the distance L over which there is significant change in A_μ. This means that there is no particular scale on which we could expect the solutions of (4) to differ qualitatively from those of the Maxwell system. (There might be exceptions to this statement among cases in which A_μ is null, so that K is infinite.)

Note that there are theories, such as that of Born and Infeld (1934), in which there are significant deviations from Maxwell's theory at very *small* distances. The parameter characterizing the deviation in the Born–Infeld theory is a characteristic field strength, which would be expected to be on the order of that at the surface of a classical electron. Thus, the theory defines a length scale on the order of the classical electron radius, and non-Maxwellian effects could be expected only at distances comparable with that length or smaller. The Proca system differs from that of Maxwell only on a sufficiently *large* scale defined by the photon rest mass. The novelty of the system (4) in comparison with these well-known theories is that the parameter characterizing deviations from Maxwellian electrodynamics [α in (2) or β in (4)] is *dimensionless*, so that (as in Maxwell's theory) there is no constant length scale picked out.

A specific situation of considerable interest is that of plane wave propagation. Since the usual electromagnetic plane waves satisfying $F^{\mu\nu}_{,\nu} = 0$ have $F_{\sigma\tau}F^{\sigma\tau} = 0$ throughout space-time (and a gauge can be chosen so that A_μ is not null), we see that such waves will satisfy (4) as well. Thus, there are plane waves which propagate without dispersion in this theory, even though Coulomb's law does not hold at any length scale. Of course, there will be other, more complex, solutions to the new equations as well.

[The fact that plane waves satisfy (4) is an extension of a result proved earlier for a large class of Lorentz- and gauge-invariant theories (Murphy, 1978). It means that this new system of equations also will not give a cosmological "tired light" effect.]

The system of field equations introduced here is not put forward as a serious alternative to the usual Maxwell theory. (Among other things, the fact that it has not been possible to find a Lagrangian for the system counts against it.) The point is rather to demonstrate that one must exercise some care in analyzing observations to test the validity of conventional electrodynamics. There is an infinite number of ways to parametrize a theory used to analyze such observations, and these are not all equivalent. For example, one might *in principle* attempt to detect a time delay between different spectral components of radiation emitted from a distant galaxy. If such a delay were found, it could be attributed to the vacuum dispersion produced by a photon rest mass and described by (1). If no such delay were detected, one could place an upper limit on the photon rest mass *in the Proca theory.* But no limit at all would be placed upon the parameter β in the system (4). In order to measure, or to place limits upon, β in this quite different theory, other experiments, such as the classic one of Plimpton and Lawton (1936), have to be appealed to.

There is no unique covering theory for Maxwellian electrodynamics. This should be kept in mind even as one uses the most plausible covering theories for comparison with observations.

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